$-\pi \leq x \leq \pi$. Eq. (7) states that for a given number $2N+1$ of Fourier terms in the expansion of $g(x)$ the function $f(u)$ may be modified from a sin $\pi u/\pi u$ form by rearranging $2N$ of the zeroes of this latter function in an arbitrary manner. The synthesis procedure is now reduced to that of choosing a suitable polynomial $P(u)$. The particular case of approximating a Chebyshev behavior for $f(u)$ is discussed by Collin[2] and Taylor.[3] Further applications to transmission line taper synthesis will be presented in a later paper. Finally, the antenna optimization problem is discussed in greater detail elsewhere.[4]

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[4] R. E. Collin and S. Rothschild, "Reactive Energy in Aperture Fields and Aperture Q," to be presented at the Copenhagen Electromagnetic Symposium, June, 1962.

A Proposed Lossless Electronic Phase Shifter*

Modern antenna array designs frequently require the use of high-speed, electronically controlled phase shifters. The use of low-noise receivers now becoming available also dictates the requirement that such phase shifters have zero, or very small, losses. The use of parametric devices as active elements in such phase shifters is particularly attractive because of their high degree of reliability, high frequency capability, and excellent noise performance. This memorandum proposes such an active electronic phase shifter, which accomplishes the additional function of providing amplification or attenuation independently of its phase shift.

Although most parametric amplifiers have been designed to amplify an incoming signal directly without frequency conversion, it is possible to design low-noise parametric up-converters with the output frequency higher than the input frequency. The reverse procedure of down-converting is not practical because the power gain is always less than unity. There are two distinct modes of operation for the parametric up-converter. The output frequency may be either the sum or the difference of the pump and signal frequencies. When the lower idler is utilized ($\omega_p = \omega_i + \omega_s$), the amplifier is a negative resistance device with unlimited gain and is potentially unstable. However, when the upper idler is employed ($\omega_i = \omega_p + \omega_s$), the up-converter is stable but the gain is limited to the ratio of the idler frequency and signal frequency.

Changes in pump frequency appear as changes in the output frequency unless the idler is down-converted in a conventional crystal mixer which uses the same pump source for the local oscillator. In this case the original input frequency is recovered. Although the output frequency of the parametric up-converter/down-converter combination is independent of pump frequency the phase shift through the combination is a function of pump frequency.

It has been shown that the expression relating the phase terms in the expression for the voltages at the three different frequencies is similar to the expression relating the frequencies with an additional phase shift of $-\pi/2$ radians due to the capacitive reactance of the varactor.[1] The expression for the phase shift in a crystal mixer does not contain the term $-\pi/2$. The phase shift in a lower sideband up-converter is given by

$$\phi_p = \phi_s + \phi_i - \frac{\pi}{2}$$

where $\omega_p = \omega_i + \omega_c$, while the phase shift in a crystal mixer is given by

$$\phi_p = \phi_s + \phi_i.$$

With the aid of Fig. 1 the following set of equations may be written:

$$\phi_{s_0} = \phi_{p_2} - \phi_{i_2}$$

$$\phi_{p_2} = -\frac{2\pi l_2}{\lambda_p}$$

$$\phi_{i_2} = \phi_{i_1} - \frac{2\pi l_3}{\lambda_i}$$

$$\phi_{i_1} = \phi_{p_1} - \phi_{s_i} - \frac{\pi}{2}$$

$$\phi_{p_1} = -\frac{2\pi l_1}{\lambda_p}.$$

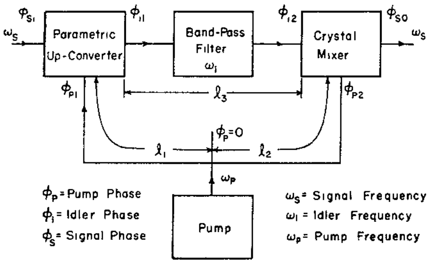

Fig. 1—Block diagram of parametric up-converter/down-converter system.

The solution for the total phase shift is

$$\phi = \phi_{s_0} - \phi_s.$$
$$= \frac{2\pi}{\lambda_p}(l_1 - l_2) + \frac{\pi}{2} + \frac{2\pi l_3}{\lambda_i}.$$

By similar manipulation, the phase shift of the upper sideband up-converter/down-converter combination is

$$\phi = \phi_{s_0} - \phi_{s_i}$$
$$= \frac{2\pi}{\lambda_p}(l_2 - l_1) - \frac{\pi}{2} - \frac{2\pi l_3}{\lambda_i},$$

which is the negative of the previous expression.

Thus, for either the lower sideband or the upper sideband up-converter/down-converter combination, the total phase shift at a given signal frequency is a function only of the pump frequency for fixed line lengths. The choice between an upper- or lower-sideband up-converter will be affected by gain and stability requirements.

Since the rate of change of phase shift vs pump frequency is controlled by the line lengths, it is possible to design for any specified range of phase shift, 360° for example, over a fixed range of the tunable pump source. This tuning range is determined by the bandwidth of the band-pass filter, which in turn is dictated by the signal frequency.

For example, assuming a signal frequency of 200 Mc and an X-band pump, a 200-Mc band-pass filter centered on 9800 Mc would support the lower idler frequency for pump frequencies between 9900 Mc and 10,100 Mc, while suppressing the upper idler. With this set of conditions, a difference of electrical line lengths between l_2 and $l_1 + l_3$ of 1.5 meters would be required to give 360° phase shift over the operating range.

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D. B. Anderson, and J. C. Aukland, "Transmission Phase Relations of Four-Frequency Parametric Devices," presented at IRE Microwave Theory and Techniques Nat'l Symp., Washington, D. C.; May 15–17, 1961.

Oversize Waveguide Directional Coupler*

Oversize rectangular waveguide using TE_{10} mode propagation has been suggested as a means of reducing the attenuation and fabrication difficulties of transmission lines and components for frequencies above 40 Gc.[1,2] Experiments on straight sections of such transmission lines show that it is superior to pure optical transmission for short distances.[3]

A directional coupler using oversize rectangular waveguide has been constructed; experimental results are described in this communication. The initial device was de-

* Received May 1, 1962.

[1] A. F. Harvey, "Optical techniques at microwave frequencies," *Proc. IEE*, vol. 106, pt. C, pp. 141–157; March, 1959.

[2] L. Lewin, "A Note on Quasi-Optical Methods at Millimeter Wavelengths," *PBI-MRI Symp. on Millimeter Waves*, Interscience Publishers, New York, N. Y., p. 469; 1959.

[3] R. H. Garnham, "Optical and Quasi-Optical Transmission Techniques and Component Systems for Millimeter Wavelengths," Royal Radar Establishment, Malverne, England, RRE Rept. No. 3020; March, 1958.

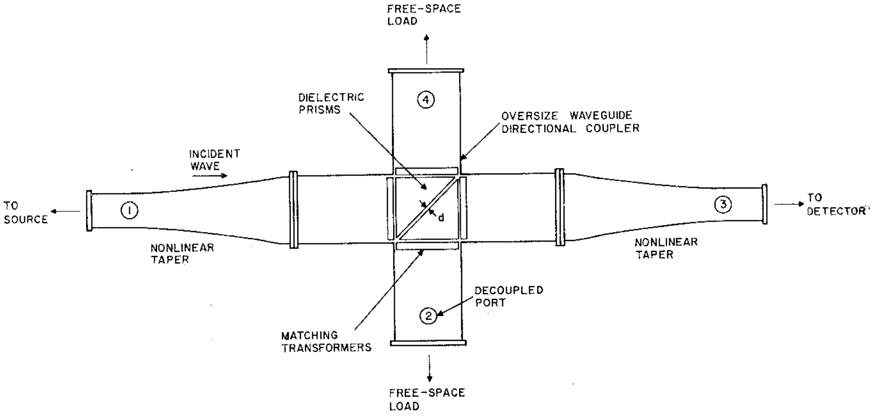

Fig. 1—Oversize waveguide directional coupler schematic.

signed for the 10-cm range; a millimeter wavelength version can be designed by direct scaling.

Fig. 1 shows the construction of the coupler including nonlinear tapers[4] used to launch a pure TE_{10} mode in the oversize guide. Fig. 2 is a picture of the device showing the dielectric coupling elements. The oversize guide had *a* and *b* dimensions of 8.51 and 5.82 inches respectively. This guide is about 3 times oversize for standard *S*-band. The coupler consists of a pair of dielectric prisms ($\epsilon_r = 2.55$) separated by an air space of thickness *d*. Quarter-wavelength dielectric slabs ($\epsilon_r = 1.6$) were used to match the prisms to the four waveguide ports. The theory follows an analysis for a pair of optical prisms.[1] Power incident at port 1 is coupled to ports 3 and 4, where the relative division is a function of the spacing between the two prisms. Port 2 is isolated from port 1. The theoretical expressions for power coupling to ports 4 and 3 (assuming a lossless dielectric) are

$$\frac{P_4}{P_1} = \frac{2.4 \sinh^2 (3.29\, d/\lambda)}{\sinh^2 (3.29\, d/\lambda) + 1.40 \cosh^2 (3.29\, d/\lambda)}$$

and

$$\frac{P_3}{P_1} = 1 - \frac{P_4}{P_1}$$

where

P_1 = power available at port 1,
P_4 = power coupled to port 4,
P_3 = power coupled to port 3,
λ = free-space wavelength.

We have constructed a 3-db directional coupler wherein the coupling to ports 3 and 4 is equal within 0.3 db and the insertion loss is about 0.8 db over a 15-per cent band centered about 2.9 Gc. The results are shown in Fig. 3. This loss was mainly due to a 0.20-db loss in the tapers, a 0.30-db dielectric loss, and wall losses. The power coupled to port 2 is 21 to 26 db down from the incident level over the band. The input SWR is about 1.3 over the band. Additional experimentation at different *d*'s agree with the theory.

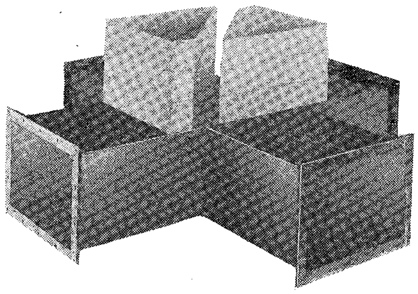

Fig. 2—Oversize waveguide directional coupler showing coupling prisms.

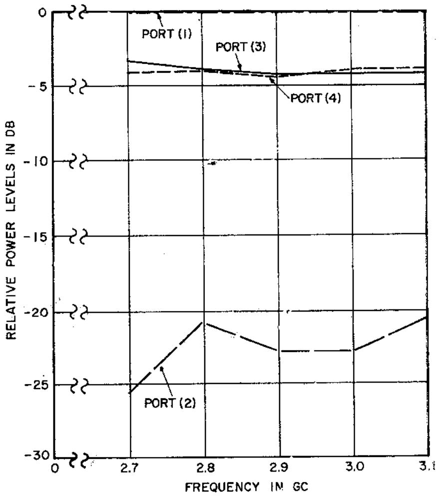

Fig. 3—Frequency response of over-size directional coupler.

Further work is in progress to scale this device to the millimeter region and to reduce the wall loss due to higher mode conversion. A device of this type can be used as an adjustable directional coupler and variable attenuator by mechanically varying *d*. By setting *d* for an equal power split, it becomes a 90° hybrid and thus can be used to form other components such as variable phase shifters and mixers.

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[4] H. G. Unger, "Circular waveguide taper of improved design," *Bell Sys. Tech. J.*, vol. 37, pp. 899–912; July, 1958.

Resonances in a Cylindrical Plasma Column*

A plasma column irradiated by an electromagnetic wave which has its electric vector and direction of propagation perpendicular to the axis of the tube exhibits Tonks-Dattner[1,2] resonances in its absorption pattern when the electron density in the column is varied. The pattern consists of a main resonance and a number of less pronounced subsidiary resonances at currents corresponding to electron densities lower than that of the main peak. The subsidiary resonances grow progressively smaller as the electron density decreases.

Herlofson[3] predicted a single resonance for a cylindrical plasma irradiated in this fashion. He treated the problem by solving the wave equation in cylindrical coordinates subject to appropriate boundary conditions, and found only one frequency for maximum scattering from the column. No resonances at all were predicted for the parallel mode of excitation in which the *E* vector is parallel to the axis of the column. So far no completely satisfactory theory has been put forward to account for the subsidiary resonances.[4,5]

In the course of experiments attempting to elucidate this phenomenon, our observations have shown that these resonances may also be seen when the electric field is parallel to the axis of the plasma column. Furthermore, if the plasma column is placed with its axis parallel to the direction of propagation in a waveguide or transmission line and the length of the column is of the order of a wavelength and can be no longer considered as a localized discontinuity, the Tonks-Dattner resonances are still clearly shown. The following experimental results illustrate these points:

Fig. 1(a) shows the normal Dattner experiment with the plasma column across a waveguide propagating a TE_{01} mode, together with the resulting absorption pattern when the current through the plasma tube is varied.

Fig. 1(b) shows the waveguide rotated through 90° with respect to the plasma column as well as the absorption pattern which again shows the Tonks-Dattner resonances.

Fig. 2(a) shows a plasma column placed across a parallel plate transmission line and a photograph of the transmission down the line as the current in the tube is varied. Above a certain current the transmission is cut off at current levels; below this we have the characteristic Tonks-Dattner resonances.

* Received April 23, 1962. This work was supported in part by a grant from the National Science Foundation.

[1] L. Tonks, "The high frequency behaviour of a plasma," *Phys. Rev.*, vol. 37, pp. 1458–1483; June, 1931.

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[3] N. Herlofson, "Plasma resonance in ionospheric irregularities," *Arkiv Fysik*, vol. 3, no. 15, pp. 247–297; 1951.

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[5] R. W. Gould, "Experiments on plasma oscillations," *Proc. Conf. on Plasma Oscillations*, Spencer, Ind., pp. 167–206; June, 1959.